STUDY OF TRANSIENT STABILITY BY TRANSIENT ENERGY FUNCTION

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ABSTRACT

Stability analysis programs are a primary tool used by power system planning and operating engineers to predict the response of the system to various disturbances. Important conclusions and decisions are made based on the results of stability studies. The conventional method of analyzing stability is to calculate the transient behaviour of generators due to a given disturbance. Direct methods of stability analysis identify whether or not the system will remain stable once the disturbance is removed by comparing it with a calculated threshold value. Direct methods not only avoid the time consuming solutions required in the conventional method, but also provide a quantitative measure of the degree of system stability. This additional information makes direct methods very attractive when the relative stability of different plans must be compared or when stability limits must be calculated quickly. Direct methods of transient stability analysis of a multi machine power system, using a function describing the system's transient energy, are discussed. By examining the trajectory of the disturbed system, the following fundamental questions are dealt with: the concept of a controlling unstable equilibrium point (U.E.P), the manner in which some generators tend to lose synchronism, and identifying the energy directly responsible for system separation. Resolving this issue will substantially improve transient stability analysis by direct method.

KEYWORDS

Transient stability, power system, directs method, transient energy function, energy merging, and Stability limit formulation.

1. INTRODUCTION

Transient stability analysis of a multi-machine power system is mainly performed by simulations and MATLAB tools. For a given initial operating condition and a specified large disturbance or sequence of disturbances, a time solution is obtained for the generators' rotor angles, speeds, powers, terminal voltages, etc. By examining the generators' rotor angles at various instants, separation of one or more generators from the rest of the system, indicating loss of synchronism, is detected. While the magnitude of the computational effort involved depends on the complexity

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of the mathematical model used, only numerical methods can be used to obtain the time solution. Even for a small power network, and with the simplest mathematical model possible, this method is slow and cumber some. For many years there has been a great deal of interest in using direct methods for transient stability analysis. These methods have in common the following general approach:

- Development of a special function by which the stability characteristics of the systems post disturbance equilibrium point is examined.
- Determination of the region of stability. This has the following practical significance: if at the beginning of last transient (e.g., at the instant of fault clearing) inside that region, the system will be stable.

Both of the above areas have received a great deal of attention by investigators. For the first area, early investigators used functions that described the system energy [1, 2, and 3]. Later, Lyapunov-type functions were suggested, and more recently, energy-type functions have been used [4, 5]. The second area has been recognized as the major cause for the conservative results obtained with direct methods. In early work, the region of stability was determined by considering the unstable equilibrium point (EUP) nearest to the stable equilibrium point. Later efforts have determined the appropriate (EUP). For the particular disturbance under consideration [6, 7].

2. A CONCEPT OF TRANSIENT ENERGY

A faulted power system, at the instant of fault clearing, possesses an excess energy that must be absorbed by the network for stability to be maintained. This energy will be referred to here as transient energy. This energy is responsible for setting the synchronous for setting the synchronous machine to swing away from equilibrium. A direct method of first swing stability analysis of a multi machine power system using a function describing the system's transient energy was proposed by Athay*, etal*. The function was called Transient Energy Function (TEF). In this critical transient energy, which is, associated with the relevant unstable equilibrium points of the post fault network encountered by the disturbed system trajectory, is identified. If system transient energy at the end of fault, (i.e., at clearing) is less than this critical energy, the system is stable; otherwise it is unstable. This leads to a very fast assessment of transient stability as compared to the conventional methods. There has been a great deal of progress achieved in developing a direct method for analyzing first swing transient stability of multi-machine power system. When a disturbance occurred in a power system, the transient energy, injected into the system during the disturbance, increases and causes machine to diverge from rest of the system. When the disturbance is removed, and as machine continues to diverge from rest of the system, its kinetic energy is being converted into potential energy. This motion will continue until the initial kinetic energy is totally converted into potential energy. When this takes place, the machine will converge toward equilibrium of the system.

2.1. Transient Kinetic Energy and the Inertial Centre

One fundamental step in defining the energy contributing to system separation is the so-called inertial centre formulation of the system equations [also referred to as centre-of-angle (COA)]. The equations describing the behaviour of the synchronous machines are formulated with respect to a fictitious inertial centre (in contrast to the usual situation where the machine's equations are formulated with respect to a synchronously moving frame of reference). The importance of this formulation is in clearly focusing on the motion that tends to separate one or more machines from the rest of the system, and in removing a substantial component of the system transient energy that does not contribute to instability, namely, the energy that accelerates the inertial centre [8,9].With this formulation, the forces tending to separate some machines from the rest of the system, and the energy components associated with their motion, can be easily identified [4,5,9].

2.2. Potential Energy Surfaces and the Critical Energy

The ability of the system to absorb (or convert) the energy component that contributes to instability depends upon the following: 1) the potential energy contours or "terrain" of the post disturbance system and 2) the particular segment of this terrain traversed by the faulted trajectory. The former depends upon using a good mathematical accounting of the system energy, which describes the energy surfaces encountered by the machine rotors as they swing away from their equilibrium positions. The energy terrain, as reflected in the potential energy contours, accounts for the amount of the rotor displacement per unit of fault energy resulting from the disturbance. The second point above simply recognizes that those energy surfaces have higher ridges in some segments than in others, and thus, the amount of rotor motion (and the corresponding energy absorbed) necessary to reach instability will vary from one trajectory to another. If the system trajectory moves in a segment of higher potential energy values, the network's capacity to absorb (and convert) the initial excess transient energy is greater, and hence, it can withstand a greater initial disturbance. On the other hand, if the faulted trajectory moves in a region where the potential energy surfaces are "shallow," the network's ability to absorb excess transient energy is much reduced, and instability occurs with a smaller disturbance. Thus, the faulted trajectory is analogous to a particle "climbing up" the potential energy "hills" around this valley. In some directions the ridge, or the peak of the hill, is higher than others. The ridge of the potential energy surface contours is called the Principal Energy Boundary Surface (PEBS) [5, 10]. This ridge has several "humps" and "saddle points." These are the so-called (U.E.P)'s which are connected by the PEBS.

For the first swing transient, the U.E.P. closest to the trajectory of the disturbed system is the one that decides the transient stability of the system. This is called the controlling (or relevant) U.E.P. for this trajectory Thus; the critical transient system energy is that which corresponds to the energy of the closest or controlling U.E.P. To complete the picture, we mention that if the system is faulted and the fault is cleared before the critical clearing time, t, the system trajectory peaks before reaching the “ridge” of the potential energy surface contours or the relevant U.E.P. At a clearing time exceeding t, the ridge is crossed (usually at some point other than the U.E.P.) and stability is lost. There is only one critical trajectory that can actually go through the controlling U.E.P.

2.3. The Transient Energy function method

In the following discussion, the mathematical model describing the transient power system behaviour is the classical model: generators represented by constant voltage behind transient reactance, and loads are modelled constant impedance.

For an n-generator system with rotor angles δ_i and inertia constants M_i, i = 1,2,..., n, the position δ_0 and speed ω_o of the inertial centre are given by,

$$\delta_0 = \frac{1}{M_T}\sum_i^n M_i\,\delta_i\ ,\ i = 1,2,...........n.$$

$$\omega_0 = \frac{1}{M_T}\sum_i^n M_i\,\omega_i \qquad ---------- \ (1)$$

Where, $M_T = \sum_{i=1}^n M_i,\ \omega_i = \dot{\delta}_i$ (i= 1,2.......n).

The rotor angles and speeds with respect to the inertial centre are defined as

$\theta_i = \delta_i - \delta_o$

$\tilde{\omega} = \omega_i - \omega_o \quad i = 1,2, \ldots, n$ .... (2)

When the system is reduced to a network only composed of fictitious buses of generator internal voltage, the generator dynamics are governed by the following equations in inertial centre frame

$\theta_i = \tilde{\omega}_i \quad$ **i = 1, 2, n** .... (3A)

$M_i \;\dot{\tilde{\omega}}_i = P_i - P_{ei} - \frac{Mi}{MT} P_{coi} \quad i = 1,2,3 \ldots, n$ .... (3B)

E_i : internal voltage behind transient reactance of generator 'i',
$Y_{ii} = G_{ii} + j B_{ii}$: Driving point admittance for internal node of generator *'i'*.
$Yij = Gij + jBij$: Transfer admittance between internal node of generator *'i'* and *'j'*,
P_{mi} : Mechanical input power of generator *'i'* .

And let $\theta_{ij} = \theta_i - \theta_j$, $(i,j = 1, 2, \ldots, n)$,
$C_{ij} = E_i E_j B_{ij}$, $D_{ij} = E_i E_j G_{ij}$ $(i, j = 1,2, \ldots, n)$,
$P_i = P_{mi} - E^2 G_{ij}$ $(i=1,2, \ldots, n)$,
$P_{ei} = \sum_{i=1}^{n}(C_{ij} \sin\theta_{ij} + D_{ij}\cos\theta_{ij})$ $(i = 1,2 \ldots, n)$,
$P_{coi} = \sum_{i=1}^{n}(P_i - P_{ei})$

By the first integration of real power mismatch in (3A)-(3B) the system energy function V can be written as

$$V = \sum_{i=1}^{n}\frac{1}{2}Mi\,\tilde{\omega}^2 - \sum_{i=1}^{n}P_i(\theta_i - \theta_i^s) - \sum_{i=1}^{n-1}\sum_{j=i+1}^{n}[C_{ij}(\cos\theta_{ij} - \cos\theta_{ij}^s) - \int_{\theta_i^s+\theta_j^s}^{\theta i+\theta j} D_{ij}\cos\theta_{ij}\, d(\theta_i + \theta_j)) \quad \ldots\ldots(4)$$

Each term in **(4)** has its clear physical meaning. And the last term in **(4)** is named dissipation energy, which is trajectory dependent and can be calculated only if the system trajectory is known. By a linear approximation of the projection of the system trajectory to angle space, this term caused by transfer
Conductance is approximated by

$$I_{ij} = D_{ij}\frac{(\theta i+\theta j)-(\theta_i^s+\theta_j^s)}{(\theta_{ij} - \theta_{ij}^s)}(\sin\theta_{ij} - \sin\theta_{ij}^s) \quad \ldots\ldots(5)$$

$$V = \sum_{i=1}^{n}\frac{1}{2}Mi\,\tilde{\omega}^2 - \sum_{i=1}^{n}P_i(\theta_i - \theta_i^s) - \sum_{i=1}^{n-1}\sum_{j=i+1}^{n}[C_{ij}(\cos\theta_{ij} - \cos\theta_{ij}^s) + I_{ij}) \quad \ldots\ldots(6)$$

Where, θ_i^s or θ_j^s are stable equilibrium angles for generator '*i' and* generator *'j'*,

The system transient energy components in Eq. (6) are identifiable. The first term is the kinetic energy. The second term is position energy, which is part of the system's potential energy. The third term is the magnetic energy, which is also part of the potential energy. The fourth term is the dissipation energy, which is the energy dissipated in the network transfer conductance (which includes part of the load impedances). As is common in the literature, we will use the term "potential energy" to indicate the last three components. Examining Eq. (6) we note that at θ^s the transient energy is zero; and at the instant of fault clearing the transient energy is greater than zero. If the system is to remain stable, the kinetic energy at the beginning of the post disturbance period must be converted, at various instants along the trajectory, to other forms of energy. Thus, the excess (transient) kinetic energy must be absorbed by the network.

It is very important to know the value of θ^{s1}(pre-fault condition) and θ^{s2}(post-fault condition) to determine transient energy at the instant fault of fault clearing w .r .t θ^{s1} called V_{cl} and post-fault system called critical energy V_{cr}. w. r. t θ^{s2} post fault equilibrium point. The mathematical expression of the *transient energy*, V_{cl} *and critical energy,* V_{cr} are given as

$$V_{cl} = V \Big|_{\theta^{s1}}^{\theta^{c}} = \sum_{i=1}^{n} \frac{1}{2} Mi(\tilde{\omega}^{cl})^2 - \sum_{i=1}^{n} P_i(\theta_i^{cl} - \theta_i^{s}) - \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} [C_{ij} (\cos\theta_{ij}^{cl} - \cos \theta_{ij}^{s}) + I_{ij}^{cl}) \quad \text{....(7)}$$

$$V_{cr} = V_u = V \Big|_{\theta^{s2}}^{\theta^{u}} = - \sum_{i=1}^{n} P_i(\theta_i^{u} - \theta_i^{s}) - \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} [C_{ij} (\cos\theta_{ij}^{u} - \cos \theta_{ij}^{s}) + I_{ij}^{u}) \quad \text{....... (8)}$$

The above two energy values are both calculated with respect to the post-fault **SEP.** So the energy margin or stability index is given by

$$\Delta V = V_{cr} - V_{cl} \quad \text{.......... (9)}$$

The system is stable if the stability index is positive.

2.4. The Unstable Equilibrium Point

Correct determination of the- U.E.P, is essential for the successful use of direct methods, based on Eq. (6), for stability analysis. For the purpose of this investigation, however, it was deemed very important to make absolutely certain that the correct U.E.P, among the numerous possibilities has been found in each case. For this reason, the U.E.P, was usually confirmed by obtaining the swing curves for a near critical disturbance. The appropriate angles from these curves were then used in a Davidon-Fletcher-Powell program to obtain the U.E.P. A similar procedure is used to obtain the post disturbance equilibrium angles θ^{s2}. From the values ofθ^{u} and θ^{s2} the energy at θ^{u} and its components are calculated using Eq. (8)

3. WORK EXAMPLE

The test system used contains **4** equivalent generators and11 buses system, as shown in Fig. 1, and the generator data and initial conditions are listed in Tab. **1.** Three-phase faults at one of the lines 10-8 near bus 10 and cleared by tripping the faulted line, are studied.

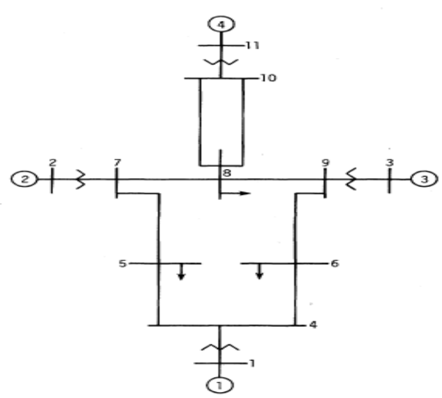

Figure 1. The 4 –generator test system.

3.1. Unstable Equilibrium Point

For the above mentioned disturbances, the relevant U.E.P's were carefully calculated 1) by the special computer program package, and 2) by starting the Davidon-Fletcher-Powell (DFP) procedure from the point where the critical generators were at the peak of their rotor angle swings and the remaining generators were at their 0s2 angles. The predicted U.E.P's and their potential energies are given below.

4 – Generator system

$\theta_1^u = -27^0$ $\theta_2^u = -11^0$

$\theta_3^u = -5.5^0$ $\theta_4^u = 11.2^0$ $V_u = 0.625$ pu

Casual examination of the above data (i.e., from the values of $\theta_i > \pi/2$) reveals which generators tend to separate from the rest of the system for the specific disturbances given. For the 4-generator system it is generator No. 4. This data is reasonable, since these generators are close to the disturbance.

3.2. System Trajectories

Stability runs, using time solutions and different clearing times, were made for the faults indicated above until the system barely went unstable. In addition to the rotor swings, information on the transient energy was obtained at different instants. It should be noted, however, that the energy is calculated with respect to, the pre-fault stable equilibrium θ^{s1}'.

Figures 3 and 4 show some of the results obtained. Machine angles (w.r.t. inertial centre) as well as the kinetic energy (K.E.) and potential energy (P.E.), are displayed for the case of t_c slightly less than the critical clearing time, and for the case where t_c was such that the system barely became unstable.

Examining Fig. 3 we note that at the peak of the swing of generator No. 4, the system potential energy is maximum and the kinetic energy is minimum (almost zero in this case). This confirms the idea of the conversion of the kinetic energy to potential energy (noting that a portion of that energy is dissipated in the transfer conductance).

Figure 4 shows that when the P.E. is maximum and the K.E. is Minimum, $\theta_4 = 112^0$ and $\theta_1 = -29°$ which are almost identical to the values predicted for θ_4^u and θ_1^u. However, the values of θ_2 and θ_3 at that instant are 1.9° and - 6.1°. They differ from the predicted values of θ_2^u and θ_3^u by a few degrees. The maximum P.E. is about 0.63 pu, and the minimum K.E., occurring at the same instant, is not exactly zero. The data shows that at critical clearing the critical machine appears to be at the position predicted by θ_4^u, while the other generators are not exactly at their U.E.P. values

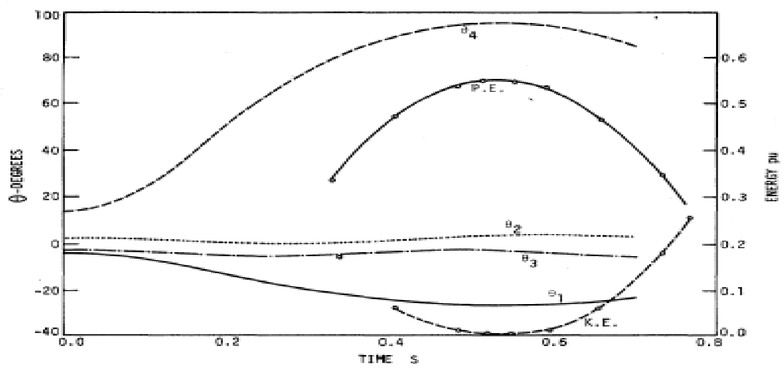

Figure 3. Four- system. Fault at Bus 10, cleared in 0.148 sec.

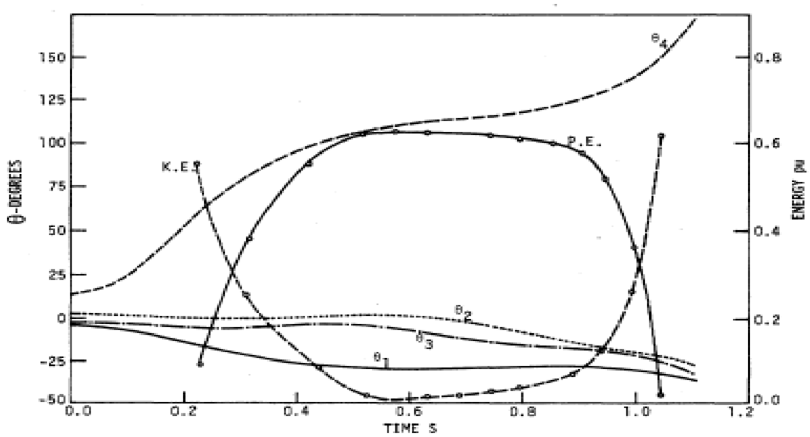

Figure 4. Four – generator system. Fault at Bus 10 cleared in 0.159 sec

4. DISCUSSION AND CONCLUSIONS

1. The results of the previous sections merit the following conclusions:
2. The concept of a controlling U.E.P. for a particular system trajectory is valid.
3. From the values of ϑ_i^u, i = 1, 2, n, the critical generators, i.e., those tending to separate from the rest, are identified by $\theta_i > \pi/2$.
4. At critical clearing the system trajectory is such that only the critical generators need pass at, or very near to, their values at the U.E.P. Other generators may be slightly off from their U.E.P. values.
5. If more than one generator tends to lose synchronism, instability is determined by the gross motion of these generators, i.e., by the motion of their centre of inertia.
6. The value of V_u (at the U.E.P.) is, for all practical purposes, equal to the critical energy, V_{cr}, for the system.
7. Not all the excess K.E. (at t_c) contributes directly to the separation of the critical generators from the rest of the system; some of that energy accounts for the other inter-generator swings. For stability analysis, that component of K.E. should be subtracted from the energy that needs to be absorbed by the system for stability to be maintained.
8. First swing transient stability analysis can be accurately made directly (without time solutions) if:

- The transient energy is calculated at the end of the disturbance, and corrected for the K.E. that does not contribute to system separation; and
- The unstable equilibrium point (θ^u) and its energy are computed.

Further research work is continuing to get a complete transient stability analysis of a large power system and voltage collapse by TEF.

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APPENDIX

Table-1; 4 - Generator System.

Gen. No.	Generator Initial		Initial Condition			
	H MWs/MVA	X_d' pu	P_m^* pu	Internal Voltage E pu	Internal Voltage δ deg.	θ^{s1} deg.
1	23.64	0.0608	2.269	1.0967	6.95	- 4.08
2	6.40	0.1198	1.600	1.1019	13.49	2.45
3	3.01	0.1813	1.000	1.1125	8.21	- 2.76
4	6.40	0.1198	1.600	1.0741	24.90	13.91

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